Integration

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2014 June C34 Q4

4. Find

(a) $\int (2x + 3)^{12} \, dx$

(2)

(b) $\int \frac{5x}{4x^2 + 1} \, dx$

(2)

2015 June C34 Q5

5. (i) Find the x coordinate of each point on the curve $y = \dfrac{x}{x+1}$, $x \neq -1$, at which the gradient is $\dfrac{1}{4}$

(4)

(ii) Given that

$$\int_{a}^{2a} \frac{t+1}{t}\,\mathrm{d}t = \ln 7 \qquad a > 0$$

find the exact value of the constant a.

(4)

2016 June C34 Q7

7.

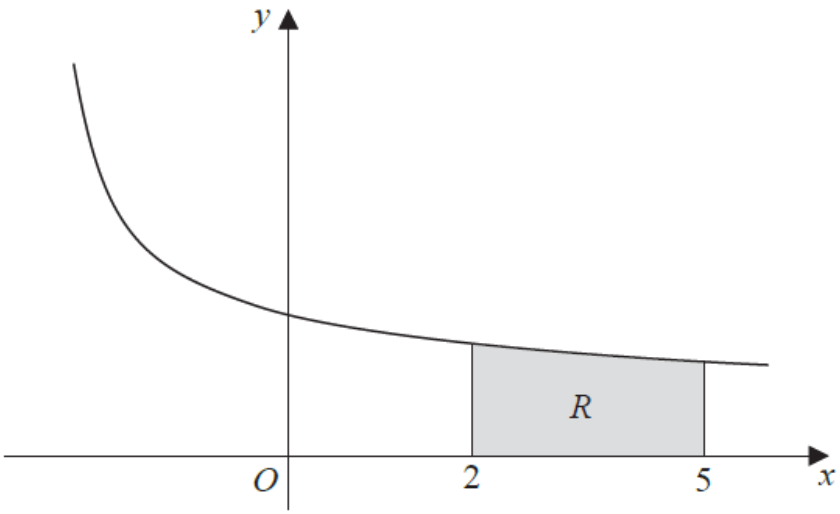

Diagram not drawn to scale

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \frac{1}{\sqrt{2x+5}}, \quad x > -2.5$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines with equations $x = 2$ and $x = 5$

(a) Use the trapezium rule with three strips of equal width to find an estimate for the area of R, giving your answer to 3 decimal places.

(4)

(b) Use calculus to find the exact area of R.

(4)

(c) Hence calculate the magnitude of the error of the estimate found in part (a), giving your answer to one significant figure.

(1)

2017 October C34 Q5

5. (i) Find

$$\int \left((3x + 5)^9 + e^{5x} \right) dx$$

(3)

(ii) Given that b is a constant greater than 2, and

$$\int_2^b \frac{x}{x^2 + 5} dx = \ln\left(\sqrt{6}\right)$$

use integration to find the value of b.

(5)

2020 January P3 Q8

8. (i) Find, using algebraic integration, the exact value of

$$\int_3^{42} \frac{2}{3x-1}\,\mathrm{d}x$$

giving your answer in simplest form.

(4)

(ii) $$\mathrm{h}(x) = \frac{2x^3 - 7x^2 + 8x + 1}{(x-1)^2} \qquad x > 1$$

Given $\mathrm{h}(x) = Ax + B + \dfrac{C}{(x-1)^2}$ where A, B and C are constants to be found, find

$$\int \mathrm{h}(x)\,\mathrm{d}x$$

(6)

2020 October P3 Q5

5. (a) Show that

$$\sin 3x \equiv 3\sin x - 4\sin^3 x$$

(4)

(b) Hence find, using algebraic integration,

$$\int_0^{\frac{\pi}{3}} \sin^3 x \, dx$$

(4)

2020 October P3 Q9

9. (a) Given that

$$\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \equiv x^2 + P + \frac{Q}{x - 4} \qquad x > -3$$

find the value of the constant P and show that $Q = 5$

(4)

The curve C has equation $y = g(x)$, where

$$g(x) = \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \qquad -3 < x < 3.5 \qquad x \in \mathbb{R}$$

(b) Find the equation of the tangent to C at the point where $x = 2$
Give your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

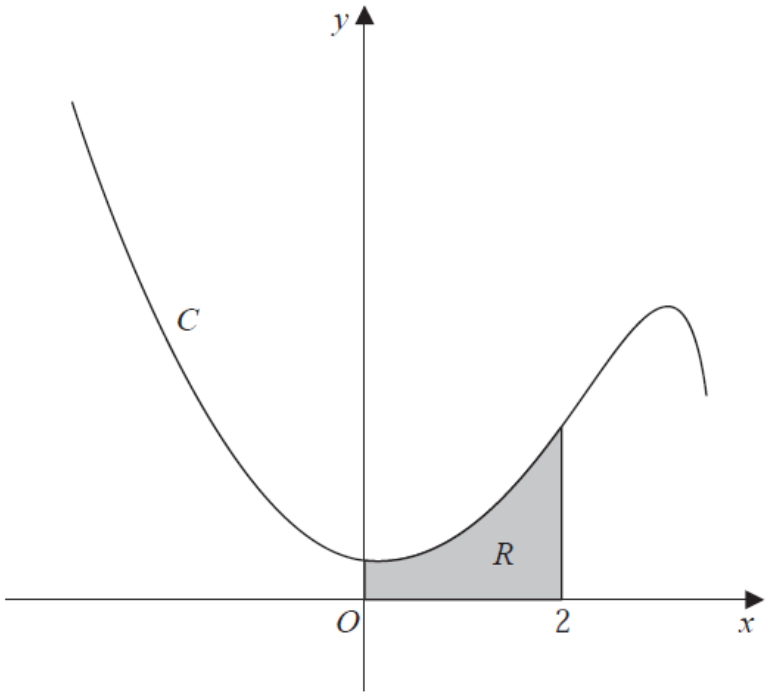

Figure 4

Figure 4 shows a sketch of the curve C.
The region R, shown shaded in Figure 4, is bounded by C, the y-axis, the x-axis and the line with equation $x = 2$

(c) Find the exact area of R, writing your answer in the form $a + b\ln 2$, where a and b are constants to be found.

(5)

2021 January P3 Q1

1. Find

$$\int \frac{x^2 - 5}{2x^3} \mathrm{d}x \qquad x > 0$$

giving your answer in simplest form.

(3)

2021 January P3 Q9

9. Find

(i) $\int \frac{3x - 2}{3x^2 - 4x + 5} \, dx$

(2)

(ii) $\int \frac{e^{2x}}{(e^{2x} - 1)^3} \, dx \qquad x \neq 0$

(2)

2021 June P3 Q3

3. (i) Find

$$\int \frac{12}{(2x-1)^2}\,dx$$

giving your answer in simplest form.

(2)

(ii) (a) Write $\dfrac{4x+3}{x+2}$ in the form

$$A + \frac{B}{x+2}$$ where A and B are constants to be found

(b) Hence find, using algebraic integration, the exact value of

$$\int_{-8}^{-5} \frac{4x+3}{x+2}\,dx$$

giving your answer in simplest form.

(6)

2021 October P3 Q5

5. (i) Find, by algebraic integration, the exact value of

$$\int_2^4 \frac{8}{(2x-3)^3}\,\mathrm{d}x$$

(4)

(ii) Find, in simplest form,

$$\int x\left(x^2+3\right)^7\,\mathrm{d}x$$

(2)

2021 October P3 Q10

10.

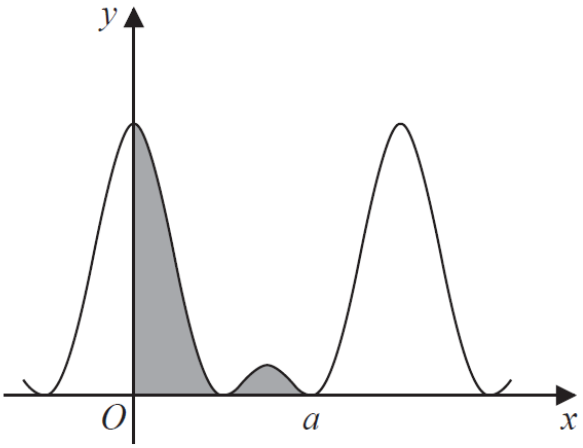

Figure 4

Figure 4 shows a sketch of part of the curve with equation

$$y = (1 + 2\cos 2x)^2$$

(a) Show that

$$(1 + 2\cos 2x)^2 \equiv p + q\cos 2x + r\cos 4x$$

where p, q and r are constants to be found.

(2)

The curve touches the positive x-axis for the second time when $x = a$, as shown in Figure 4.

The regions bounded by the curve, the y-axis and the x-axis up to $x = a$ are shown shaded in Figure 4.

(b) Find, using algebraic integration and making your method clear, the exact total area of the shaded regions. Write your answer in simplest form.

(5)

2022 January P3 Q3

3. (i) Find, in simplest form,

$$\int (2x - 5)^7 \, dx$$

(2)

(ii) Show, by algebraic integration, that

$$\int_0^{\frac{\pi}{3}} \frac{4\sin x}{1 + 2\cos x} dx = \ln a$$

where a is a rational constant to be found.

(4)

2022 June P3 Q3

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Given that k is a positive constant,

(a) find

$$\int \frac{9x}{3x^2 + k} \, dx$$

(2)

Given also that

$$\int_2^5 \frac{9x}{3x^2 + k} \, dx = \ln 8$$

(b) find the value of k

(4)

2022 October P3 Q1

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = \frac{2x^3 - 4x - 15}{x^2 + 3x + 4}$$

(a) Show that

$$f(x) \equiv Ax + B + \frac{C(2x + 3)}{x^2 + 3x + 4}$$

where A, B and C are integers to be found.

(4)

(b) Hence, find

$$\int_3^5 f(x)\,dx$$

giving your answer in the form $p + \ln q$, where p and q are integers.

(5)

2023 January P3 Q4

4. $$\mathrm{f}(x) = \frac{2x^4 + 15x^3 + 35x^2 + 21x - 4}{(x+3)^2} \qquad x \in \mathbb{R} \quad x > -3$$

(a) Find the values of the constants A, B, C and D such that

$$\mathrm{f}(x) = Ax^2 + Bx + C + \frac{D}{(x+3)^2}$$

(4)

(b) Hence find,

$$\int \mathrm{f}(x)\,\mathrm{d}x$$

(3)

2023 January P3 Q8

8. Find, in simplest form,

$$\int (2\cos x - \sin x)^2 \, dx$$

(5)

2023 June P3 Q3

3. (i) Find $\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(\sin^2 3x\right)$ writing your answer in simplest form.

(2)

(ii) (a) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(3x^2 - 4\right)^6$

(2)

(b) Hence show that

$$\int_0^{\sqrt{2}} x\left(3x^2 - 4\right)^5 \mathrm{d}x = R$$

where R is an integer to be found.

(*Solutions relying on calculator technology are not acceptable.*)

(3)

2023 October P3 Q3

3. (a) Using the identity for $\cos(A + B)$, prove that

$$\cos 2A \equiv 2\cos^2 A - 1$$

(2)

(b) Hence, using algebraic integration, find the exact value of

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} (5 - 4\cos^2 3x)\,\mathrm{d}x$$

(4)